Supersymmetry Breaking through Boundary Conditions Associated with the $U(1)_R$

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Abstract

The effects of boundary conditions of the fields for the compactified space directions on the supersymmetric theories are discussed. The boundary conditions can be taken to be periodic up to the degrees of freedom of localized $U(1)_R$ transformations. The boundary condition breaks the supersymmery to yield universal soft supersymmetry breaking terms. The 4-dimensional supersymmetric QED with one flavour and the pure supersymmetric QCD are studied as toy models when one of the space coordinates is compactified on S^1.

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In this paper we investigate the effects of boundary conditions of fields on supersymmetry breaking when one of the space coordinates is compactified on S^1. One does not know, *a priori*, what boundary conditions should be imposed on fields for the S^1-direction. We shall consider general boundary conditions except for the periodic one. This is very contrary to the case of the finite temperature field theory, in which the boundary conditions for the euclidean time direction is determined definitely by the quantum statistics of particles.

As general and possible boundary conditions, one can require that the fields return to their original values up to phases [1] proportional to their charges of global symmetry transformations[1] when the fields travel along the S^1 direction. The global symmetry transformations must be symmetry of the theories . The lagrangian is still single-valued even if the fields have such the boundary conditions. The boundary conditions are periodic for the S^1 direction up to the degrees of freedom of local transformations. In other words, the fields with the boundary conditions are mutually related with the fields with the periodic boundary condition by transformations which are obtained by localizing the aforementioned global symmetries and whose parameters depend linearly only on the compactified coordinate [2].

Having the boundary conditions mentioned above, the translational invariance may be broken for a certain boundary condition. The translational invariance, however, crucial for the supersymmetric invariance. In supersymmetric theories the variations of action under the supersymmetric transformations vanish up to total derivative terms. If the translational invariance is broken for some compactified directions due to the boundary conditions, the total derivative terms do not vanish and remain as surface terms. The supersymmetry is explicitly broken in this case. Thus, the boundary conditions may break the supersymmetry.

One may think that we can use the boundary conditions associated with the global gauge symmetry in supersymmetric gauge theories. The boundary conditions, however, do not break the translational invariance. One can always redefine the fields so as to satisfy the periodic boundary condition by local gauge transformations whose parameters depend linearly on the compactified coordinate. Since the local gauge symmetry is the symmetry of the theory, the effects of non-trivial phases disappear and the supersymmetry is not broken.

In order to have possibilities to break the supersymmetry by boundary conditions, their charges of global symmetry transformations must be different between the bosons and the fermions in a supermultiplet. The $U(1)_R$ symmetry, which is global symmetry of the theory, is the candidate in supersymmetric theories. The total derivative terms do not return to their original values after the translation along the S^1 direction because

[1] We ignore phases for the fermion fields associated with the continuous spin structure of the manifold in this paper.

of the charge differences between the bosons and the fermions in a supermultiplet. And they remain as the surface terms. The translational invariance is, thus, broken for the S^1 direction in this case. Therefore, the supersymmetry is explicitly broken by the boundary condition.

All the effects of the boundary condition associated with the $U(1)_R$ symmetry turn out to appear as universal soft supersymmetry breaking terms. One can redefine the fields so as to satisfy the periodic boundary condition by localized $U(1)_R$ transformation. Because the $U(1)_R$ symmetry is not the local symmetry, the transformation is not respected as a symmetry of the theory by the terms including the ordinary derivative, ∂_μ. The supersymmetry breaking terms are generated only through the derivative. As the consequence, such generated terms have couplings with mass dimensions and are the soft supersymmetry breaking terms since a derivative has mass dimension one. Moreover, the derivatives are the same for all flavours, so that the generated terms are common to all flavours which may be needed to avoid the FCNC. It should be stressed that once one has the boundary condition, such desirable supersymmetry breaking terms are automatically incorporated into theory with an unique parameter, $U(1)_R$ "couplings".

Now, let us see how the boundary condition associated with the $U(1)_R$ symmetry actually breaks the supersymmetry and how the soft supersymmetry breaking terms appear. We shall study the 4-dimensional supersymmetric QED (SQED) with one flavour and the pure supersymmetric QCD (SQCD) as toy models when one of the space coordinates, say, $x^3 \equiv y$ is compactified on S^1 whose length is L.

First, let us study the SQED with no flavours. The lagrangian of the 4-dimensional supersymmetric QED is constructed by the F-term of $W_A(\theta)^2$, where the chiral spinor superfield, $W_A(\theta)$ contains the vector boson, V_μ (photon), a two-component Weyl fermion, λ_A $(A = 1, 2)$ (gaugino) and the auxiliary field, D in the Wess-Zumino gauge [3]. The θ is the superspace coordinates. Under the supersymmetric transformation, δ_ξ in uncompactified 4-dimension, the on-shell lagrangian, $\mathcal{L}_{SQED}$ varies as $\delta_\xi \mathcal{L}_{SQED} = \partial_\mu X^\mu$, where X^μ is calculated as

$$X^\mu(\xi, \lambda, V_\mu) = -\xi\sigma_\nu\bar{\lambda}V^{\mu\nu} + \frac{i}{2}\xi\sigma^{\rho\sigma}\sigma^\mu\bar{\lambda}V_{\rho\sigma} + h.c. \quad . \tag{1}$$

Here the ξ is the supersymmetric transformation parameters of two-component constant Weyl spinors and $V_{\rho\sigma}$ is the field strength for the photon.

The $U(1)_R$ symmetry is defined by $W_A(\theta) \to \mathrm{e}^{i\beta}W_A(\mathrm{e}^{-i\beta}\theta)$. It is obvious that λ has a $U(1)_R$ charge and V_μ and D are neutral under the symmetry. Using the symmetry, we define the boundary conditions of the fields for the S^1 direction as follows;

$$V_\mu(x^i, y + L) = V_\mu(x^i, y), \quad \lambda(x^i, y + L) = \mathrm{e}^{i\beta}\lambda(x^i, y). \tag{2}$$

The lagrangian we consider is still singled-valued even if we take the boundary condition, (2). The superfield, $W_A(\theta)$ itself does not return to its original values after a translation along the S^1-direction, but the F-terms of $W(\theta)^2$ by which the supersymmetric lagrangian is constructed contain only the bilinear form, $\lambda\sigma^\mu\partial_\mu\bar{\lambda}$. Therefore, the non-trivial phase, $\mathrm{e}^{i\beta}$ disappears when the lagrangian travels along the S^1-direction. Having the boundary condition (2), the surface term,

$$\delta_\xi S_{SQED} = (\int d^3\mathbf{x}\ X^3(\xi, V_\mu, \lambda)(x^i, y)) \mid_{S^1} \tag{3}$$

does not vanish because there is a difference between $X^3(x^i, y+L)$ and $X^3(x^i, y)$ due to the non-trivial phases of (2). Here the $X^3(\xi, V_\mu, \lambda)$ is the third space-component of the total derivative terms, X^μ. Note that ξ obeys the periodic boundary condition. The $x^i (i = 0, 1, 2)$ stands for uncompactified coordinates. The translational invariance for the S^1 direction is broken by the boundary condition.

When we expand the fields in the Fourier series for S^1 direction,

$$V_\mu(x^i, y) = \frac{1}{\sqrt{L}} \sum_{n=-\infty}^{+\infty} V_\mu^{(n)}(x^i)\mathrm{e}^{\frac{2\pi i}{L}ny}, \quad \lambda(x^i, y) = \frac{1}{\sqrt{L}} \sum_{n=-\infty}^{+\infty} \lambda^{(n)}(x^i)\mathrm{e}^{\frac{2\pi i}{L}(n+\frac{\beta}{2\pi})y},$$

we see that $\lambda(x^i, y)$ is always redefined so as to satisfy the periodic boundary condition by the local transformation whose parameters depend linearly only on the compactified coordinate y;

$$\lambda(x^i, y) = U_R(y)\tilde{\lambda}(x^i, y) \quad \text{with} \qquad U_R(y) \equiv \mathrm{e}^{i\frac{\beta}{L}y}, \tag{4}$$

where $\tilde{\lambda}(x^i, y)$ satisfies $\tilde{\lambda}(x^i, y+L) = \tilde{\lambda}(x^i, y)$. The relation (4) is the same with those of the coordinate dependent compactifications used in the supergravity [2] and superstring [4]. But we find that the effects of the boundary condition to supersymmetric theories are remarkable which will be discussed below.

Suppose for a moment that the $U(1)_R$ is a local symmetry of the theory. The gauge field, V_μ^R is necessarily introduced into the theory. In this case one can always redefine the field, $\lambda(x^i, y)$ so as to satisfy the periodic boundary condition by the local transformation, $U_R^\dagger(y)$ without contradicting the local symmetry of the theory. The local transformation, $U_R^\dagger(y)$, which is now a part of the local gauge transformation, shifts only the gauge field, V_3^R by the constant, β/eL. The shift is compensated by the gauge transformation of the original gauge field, V_μ^R. Here the e is a gauge coupling constant. One can say that the theory written in terms of (V_μ, λ) is equivalent to the theory written in terms of $(V_\mu, \tilde{\lambda})$ thanks to the local gauge invariance of the theory. The boundary condition does not break the translational invariance, nor the supersymmetric invariance.

But the $U(1)_R$ symmetry is not actually a local symmetry of the theory. It is impossible to make the boundary condition periodic by the local transformation, $U_R(y)$ keeping the equivalence of the theories with the two different boundary conditions.

Once we impose the boundary condition, (2), the translational invariance is broken for the S^1 direction and the supersymmetry is broken explicitly by the surface terms, (3). How do the effects of the breaking manifestly appear in the lagrangian? The supersymmetry transformations are no longer defined in terms of V_μ and λ because they do not satisfy the same boundary condition. In order to understand the supersymmetry breaking at the lagrangian level, one needs to have the supersymetry transformations. We can define the supersymmetry transformations between V_μ and the redefined field, $\tilde{\lambda}$ by forming $V_\mu(x^i, y)$ and $\tilde{\lambda}(x^i, y)$ into a supermultiplet. The two fields satisfy the same boundary condition, say, periodic boundary condition. The effects of the supersymmetry breaking can be understood as a difference from the periodic boundary condition. One can not manifest the effects of the supersymmetry breaking through the boundary condition until the supersymmetry transformation is defined in terms of the fields satisfied by the same boundary condition between the bosons and the fermions in a supermultiplet.

By redefining λ by (4) into $\tilde{\lambda}$, the variation of the lagrangian under the modified supersymmetric transformations becomes as follows;

$$\tilde{\delta_\xi}\mathcal{L}_{SQED} = \partial_\mu X^\mu(\xi, V_\mu, \tilde{\lambda}) + U_R\partial_\mu U_R^\dagger[i\xi\sigma^{\rho\sigma}\sigma^\mu\bar{\tilde{\lambda}}V_{\rho\sigma} + h.c.], \tag{5}$$

where $\tilde{\delta_\xi}$ defines the modified supersymmetric transformations in terms of V_μ and $\tilde{\lambda}$. The first term in (5) does not generate surface terms because all the fields satisfy the periodic boundary condition. The boundary condition associated with the $U(1)_R$ symmetry breaks the supersymmetry explicitly as shown in the second term in (5). As we expected, the breaking of the supersymmetry is entirely due to the locality of the $U(1)_R$ transformation, *i.e.* $U_R\partial_\mu U_R^\dagger$. If the $U(1)_R$ is a local symmetry of the theory, the second term in (5) is absorbed into the gauge field associated with the gauged $U(1)_R$ symmetry.

Let us discuss the supersymmetry breaking terms. In terms of redefined field, $\tilde{\lambda}$, the $\mathcal{L}_{SQED}$ can be rewritten as

$$\mathcal{L}_{SQED}(V_\mu, \lambda) = \mathcal{L}_{SQED}(V_\mu, \tilde{\lambda}) + \mathcal{L}_{SQED}^{soft}, \tag{6}$$

where $\mathcal{L}_{SQED}(V_\mu, \tilde{\lambda})$ is the same with the original lagrangian except that all the fields satisfy the periodic boundary condition, and $\mathcal{L}_{SQED}^{soft}$ is obtained as

$$\mathcal{L}_{SQED}^{soft} = -i[U_R(y)\partial_\mu U_R^\dagger(y)]\tilde{\lambda}\sigma^\mu\bar{\tilde{\lambda}} = -\frac{\beta}{L}\tilde{\lambda}\sigma^3\bar{\tilde{\lambda}} = -\frac{\beta}{L}(\bar{\psi}_1\psi_1 + \bar{\psi}_2\psi_2),$$

where in the last equality we have used the Majorana spinors in the 3-dimension defined by $\tilde{\lambda}_M^T = (\psi_1, i\psi_2)^T$. The λ_M is the 4-component Majorana spinors in the 4-dimension constructed by $\tilde{\lambda}_M^T \equiv (\tilde{\lambda}_A, \bar{\tilde{\lambda}}^{\dot{A}})^T$. We note that the periodicity in β is recovered by the redefinition of the gaugino field, $\lambda \to \mathrm{e}^{2\pi i n y/L}\lambda$. The $\mathcal{L}_{SQED}^{soft}$ is generated through the

derivative, ∂_μ in the kinetic term for the gaugino, which does not respect the invariance of the theory under the local transformation, $U_R(y)$. Therefore, the kinetic term is the only source for the supersymmetry breaking. As the remarkable consequence, the supersymmetry breaking terms generated in this mechanism are common to all flavours. Moreover, the breaking is always so-called soft breaking. This is because the derivative has mass dimension one, so that the couplings generated through the derivative are always dimensional couplings. Note that the second term in (5) coincides with the modified supersymmetric transformations of $\mathcal{L}^{soft}_{SQED}$. If the $U(1)_R$ is a local symmetry of the theory, the term is absorbed into the gauge field associated with the gauged $U(1)_R$ symmetry.

Let us briefly discuss effects of the boundary condition arising from a flavour multiplet on the supersymmetry breaking. The theory we consider is the SQED with one massive flavour. We introduce two massive chiral superfields, Φ_i $(i = 1, 2)$ and assign the gauge charge $+e$ for the Φ_1 and $-e$ for the Φ_2. The gauge invariant superpotential is given by $W = m\Phi_1\Phi_2$. For each chiral superfield, the Φ_i contains a complex scalar, A_i(selectron), a two-component Weyl spinors, χ_i(electron) and the auxiliary field, F_i. As discussed before, the supersymmetry is not broken by the boundary condition associated with the global gauge symmetry, which corresponds to the conservation of the lepton number. The $U(1)_R$ symmetry is only operational for its breaking. We can define the $U(1)_R$ symmetry by $\Phi_1(\theta) \to \mathrm{e}^{i\beta}\Phi_1(\mathrm{e}^{-i\beta}\theta)$ and $\Phi_2(\theta) \to \mathrm{e}^{i\beta}\Phi_2(\mathrm{e}^{-i\beta}\theta)$. It is obvious that the fermion field, χ_i does not carry the $U(1)_R$ charge and the complex scalar field, A_i and the auxiliary field, F_i carry the $U(1)_R$ charge.

The field, A_i is always redefined so as to satisfy the periodic boundary condition by the local transformation, $A_1(x^i, y) = U_R(y)\tilde{A}_1(x^i, y),\ A_2(x^i, y) = U_R(y)\tilde{A}_2(x^i, y)$, where $U_R(y) \equiv \mathrm{e}^{i\beta/L}$. The derivative in the kinetic term for A_i does not respect the invariance under the local transformation, $U_R(y)$, so that the soft supersymmetry breaking term are generated from there. Hence, the result is given by

$$\mathcal{L}^{flavour}_{SQED} = (\frac{\beta}{L})^2(\left|\tilde{A}_1\right|^2 + \left|\tilde{A}_2\right|^2) + \frac{2e\beta}{L}(\left|\tilde{A}_1\right|^2 - \left|\tilde{A}_2\right|^2)\phi,$$

where the third space-component of the gauge field, V_3 is denoted by ϕ. We find that the soft supersymmetry breaking terms are the scalar mass terms and the trilinear scalar couplings which are common to all flavours. The supersymmetry breaking terms depend on an unique parameter, β and the gauge coupling, e.

Finally, let us discuss the SQCD with the gauge group $SU(N_c)$ and no flavours. The discussions are almost the same with the case for the SQED. One can take the boundary conditions of the fields as follows [1];

$$A_\mu(x^i, y + L) = U_g A_\mu(x^i, y)U_g^\dagger, \quad \lambda(x^i, y + L) = \mathrm{e}^{i\beta}U_g\lambda(x^i, y)U_g^\dagger, \tag{7}$$

where $U_g \in SU(N_c)$. The non-trivial phase $\mathrm{e}^{i\beta}$ is associated with the $U(1)_R$ symmetry.

As for the factor, U_g associated with the global gauge symmetry, it is shown to give no physical effects at least classically, *i.e.*, the fields can always be taken to be periodic by utilizing the freedom of the local gauge transformation whose parameters depend linearly only on y. The factor, however, is actually related with the non-integrable phases of the gauge field along the S^1 direction whose effects are essential at the quantum level for studying the local gauge symmetry breaking of the theory [5]. Let us see how the softly supersymmetry breaking terms are generated by the boundary condition. In terms of the redefined fields, the lagrangian can be rewritten as

$$\mathcal{L}_{SQCD}(V_\mu, \lambda) = \mathcal{L}_{SQCD}(V_\mu, \tilde{\lambda}) + \mathcal{L}^{soft}_{SQCD},$$

where $\mathcal{L}_{SQCD}(V_\mu, \tilde{\lambda})$ is the same with the original lagrangian except that λ is replaced by $\tilde{\lambda}$ which satisfies the periodic boundary condition. The $\mathcal{L}^{soft}_{SQCD}$ is obtained as

$$\mathcal{L}^{soft}_{SQCD} = -2i[U_R(y)\partial_\mu U_R^\dagger(y)]\mathrm{tr}[\tilde{\lambda}\sigma^\mu \bar{\tilde{\lambda}}] = -\frac{2\beta}{L}\mathrm{tr}[\tilde{\lambda}\sigma^3 \bar{\tilde{\lambda}}] = -\frac{2\beta}{L}\mathrm{tr}(\bar{\psi}_1\psi_1 + \bar{\psi}_2\psi_2),$$

where we have used the definition of the 3-dimensional Majorana spinors as before. Here again, one can conclude that the soft supersymmetry breaking terms are generated through the kinetic term of the gaugino.

We have discussed that the supersymmery can be broken explicitly by the boundary conditions associated with the $U(1)_R$ symmetry. The $U(1)_R$ charges are different between the bosons and fermions in a supermultiplet. Then, the translational invariance for the S^1 direction is broken by the boundary condition, so that the surface terms of the total derivative remain. One obtains all the effects of the boundary condition as the soft supersymmetry breaking terms in the lagrangian. This can be shown explicitly by redefining the fields so as to satisfy the periodic boundary condition by the local transformation, (4). The effects are always soft supersymmetry breaking because they are generated only through the derivative, ∂_μ, which has mass dimension one, in the kinetic term for the gaugino and selectron, where the local transformation, $U_R(y)$ is not respected as the symmetry of the theory. Remarkable feature is that the soft supersymmetry breaking terms do not have many arbitrary parameters, as usually discussed, but they depend on an unique parameter, β and the gauge coupling. These soft supersymmetry breaking terms are common to all flavours because the terms are generated from the same origin, that is, the derivative in the kinetic term. It is desirable to avoid the FCNC. It should be stressed that these soft supersymmetry breaking terms are automatically incorporated into the theories by the boundary conditions associated with the $U(1)_R$ symmetry.

The boundary conditions, U_g in (7) are related with the non-integrable phases of the gauge fields along the S^1 direction. One can study how the local gauge symmetry of the theory with the soft supersymmetry breaking terms discussed here is spontaneously

broken through the dynamics of the non-integrable phases [5]. We believe that there are new possibilities for building models with soft supersymmery breaking terms in more realistic higher dimensional super Yang-Mills theories [6]. These issues are under investigations and will appear soon.

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